Műhelytanulmányok

Vállalatgazdaságtan Tanszék

1053 Budapest, Veres Pálné u. 36., 1828 Budapest, Pf. 489
(+36 1) 482-5901, fax: 482-5844, www.uni-corvinus.hu/vallgazd

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Dobos Imre

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Budapesti Közgazdaságtudományi és Államigazgatási Egyetem
Vállalatgazdaságtan Tanszék
Veres Pálné u. 36.
H-1053 Budapest
Hungary

The generalization of Schrady´s model: a model with repair

Imre Dobos
Budapest University of Economics and Public Administration
Veres Pálné u. 36
H-1053 Budapest
Hungary

Abstract

The aim of the paper is to investigate a reverse logistics model. Schrady (1967) has examined a model with two stores, where the returned items are repaired then used as new one. The not repairable items are disposed of and the required products are procured. The author has analyzed the model with one procurement batch and two or more repair batches. We generalize this model and it is allowed more than one procurement batches. We show that the inventory holding strategy offered by Schrady is not always optimal and the policy offered by us gives a better result as that of Schrady.

Keywords: EOQ model, Production, Recycling, Waste disposal, Cost minimization

Összefoglalás

Schrady (1967) egy javítható temékű reverz logisztikai rendstert vizsgált. A termékek a felhasználótól visszakerülnek a karbantartásra, ahol azokat kijavítják, és mint újat újra felhasználják. A kereslet a termék iránt időben konstans. Nem minden termék kerül vissza a karbantartáshoz, ezért a hiányzó mennyiséget beszerzik, de csak egy tétel segítségével. A cél a rendelési fixköltség és a készlettartási költségek minimalizásása. A dolgozatban az alapmodellt terjesztjük ki arra az esetre, ha több, mint egy beszerzési tételt engedünk meg.

Kulcsszavak: Tételnagyság modell, Termelés, Újrafelhasználás, Hulladékkezelés, Költségminimalizálás

1. Introduction

A deterministic EOQ-type inventory model for reparaible items was first offered by Schrady (1967). This model can be seen as the first reverse logistics model. His model has examined the U.S. Naval Supply Systems Command stock holding problem with reparaible items. The reparaible items may be scrapped upon a failure, but the products are usualy returned from the user to the overhaul and repair point. The repaired items are sent then to the ready-for-issue (RFI) inventory to await demand. Based on the feasibility of repair, the items not sent back are disposed of and they are replaced with new procured products. The from user returned and not repaired items are held in a second stock point, i.e. the inventory of non-ready-for-issue (NRFI) items are awaiting repair at the overhaul and repair point.

Schrady has offered to inventory holding policy to solve problem: the “continuous supplement” and “substitution” policies. To this last policy he has determined the optimal procurement and repair quantites. It was assumed that there are only one procurement quantity (batch size) and more than one repair quantities.

The aim of the paper is to analyze the introduced substitution policy in a general framework. In this generalization it is allowed a more than one procurement quantity. To solve the problem we use the meta-model [1]. Schrady has not investigated the integer solution for the repair batch number, it is examined now. We will show that the by Schrady offered solution can be improved in dependence on the recovery (return) rate.

The paper is organized as follows. The next section summarizes the parameters and funtioning of the model. In section 3. we construct the inventory holding cost function of the model. Then analyzing the total average costs, we determine the optimal procurement/repair cycle. After eliminating the cycle time we have attained the model in dependence on procurement and repair batch numbers which leads to the meta-model investigated by the author, as well. Section 5. presents the basic model of Schrady with one procurement batch. We will show the optimal integer solution to this model. The following section solves the generalized model with continuous batch numbers.

2. Parameters and functioning of the model

The system contains two inventories. The user’s demand can be satisfied from the RFI inventory. The demand of the user is constant in time. The RFI inventory is filled up with procured and repaired items. Shortage is not allowed in this stock point. The procurement and repair quantities are equal. From the user the reparaible items are sent back to the overhaul and repair point with a constant rate. The reparaible items are stored in the NRFI stock point awaiting for repair. After repair products are seen as new and they are sent back to the RFI inventory. The material flow of the model is depicted in Figure 1. We define the variables and parameters as follows:

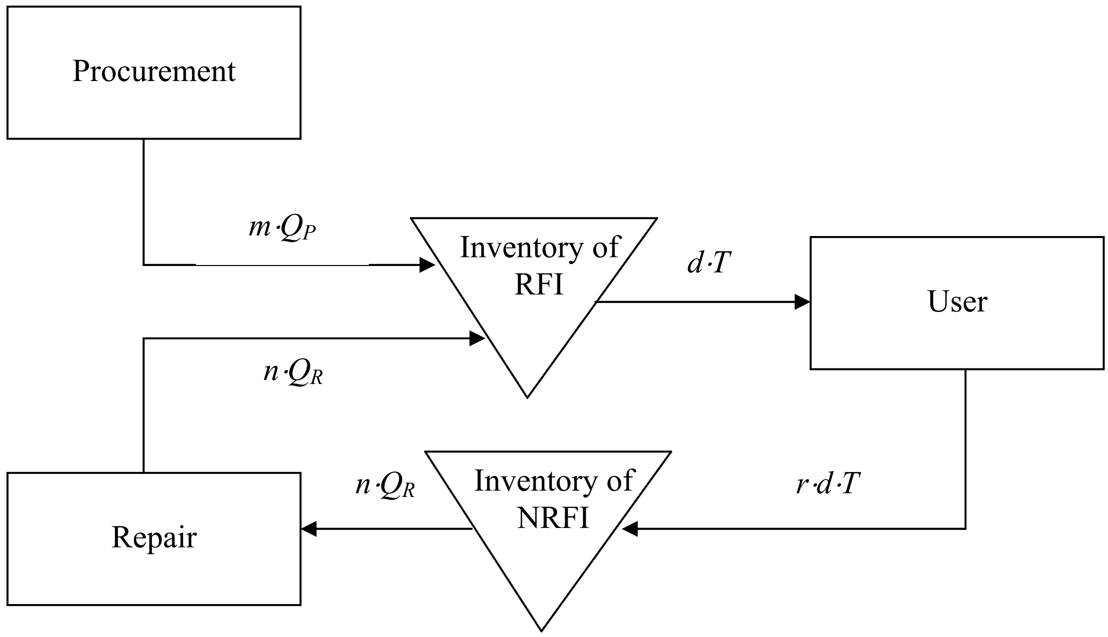
The decision variables of the model:

- Q_P procurement quantity,
- m number of procurements, $m \geq 1$, integer,
- Q_R repair batch size,
- n number of repair batches, $n \geq 1$, integer,
- T procurement/repair cycle time.

Parameters of the model:

- d demand rate, units per unit time,
- r the recovery (return) rate, percent of the demand rate d, the scrap rate is $1-r$,
- A_P fixed procurement cost, per order,
- A_R fixed repair batch induction cost, per batch,
- h_1 RFI holding cost, per unit per time,
- h_2 NRFI holding cost, per unit per time.

Figure 1. Material flow of the model

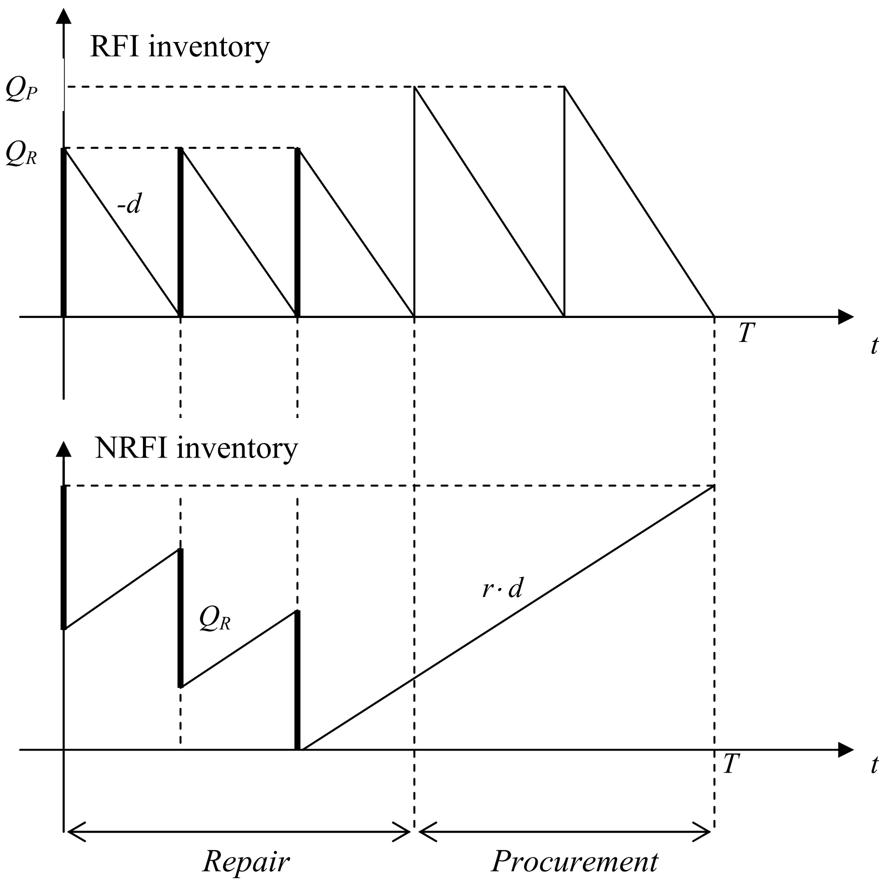


The following equalities show relations between the in- and outflows in the stocking points in a procurement/repair cycle.

$$\begin{aligned} m \cdot Q_P + n \cdot Q_R &= d \cdot T \\ n \cdot Q_R &= r \cdot d \cdot T \end{aligned} \tag{1}$$

The offered "substitution" policy has the next property. The lead times for procurement and repair batches are disregarded, because in deterministic models its influence can be eliminated with a moving away. Let us assume that a procurement/repair cycle begins with induction of a repair cycle. The initial inventory level in NRFI stock point is reduced with a repair batch size. Then the remaining NRFI inventory decreases with a new repair batch, until it reaches the zero inventory level after supply in the RFI inventory. The time history of this policy is shown in Figure 3.

Figure 2. Inventory levels in the RFI and NRFI stock points (*n = 3*, *m = 2*)



In the next two sections we construct the inventory holding and average inventory cost function of the model.

3. The inventory holding cost function

The holding costs of the model are calculated with the help of the inventory levels in time, as it is presented on Figure 2.

Lemma 1.

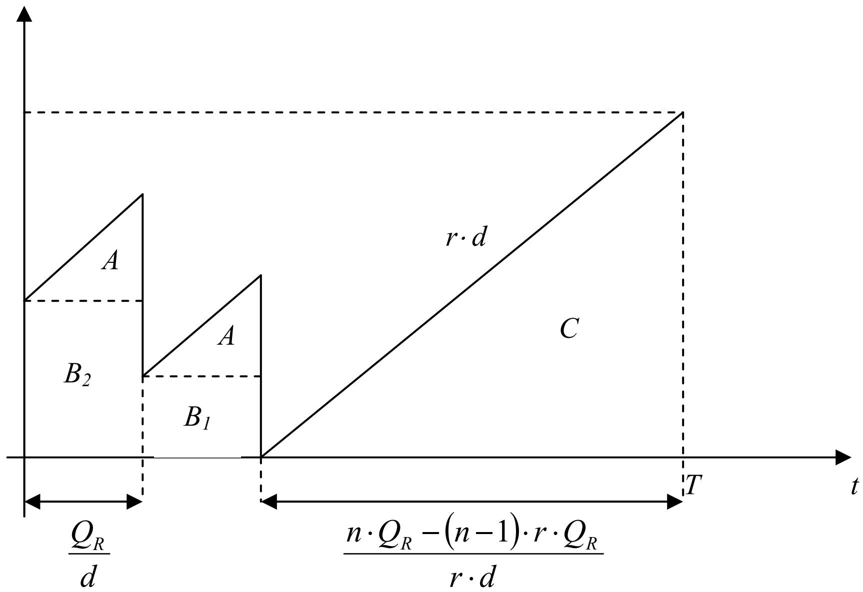
Let the inventory holding costs for RFI items H_{RFI} and for NRFI items H_{NRFI}. Then the cost functions has the next form:

$$H_{RFI} = \frac{h_1}{2 \cdot d} \cdot m \cdot Q_P^2 + \frac{h_1}{2 \cdot d} \cdot n \cdot Q_R^2$$

$$H_{NRFI} = \frac{h_2}{2 \cdot d} \cdot \left(n^2 \cdot \frac{1-r}{r} + n \right) \cdot Q_R^2$$

Proof. We will prove the second equation for the NRFI items, the first equation can be calculated in a similar way. Let us divide the area into *n-1* triangles *A*, triangle *C* and *n-1* rectangles B_1, B_2, ..., B_{n-1}. See Figure 3. The length of a repair cycle is $\frac{Q_R}{d}$. The area of a triangle A is $\frac{1}{2}\cdot r\cdot Q_R\cdot\frac{Q_R}{d}$. The area of a rectangle B_i is equal to $i\cdot(1-r)\cdot Q_R\cdot\frac{Q_R}{d}$. The maximum inventory level of NRFI items is $n\cdot Q_R-(n-1)\cdot r\cdot Q_R$. The area of triangle C is $\frac{1}{2}\cdot[n\cdot Q_R-(n-1)\cdot r\cdot Q_R]\cdot\frac{n\cdot Q_R-(n-1)\cdot r\cdot Q_R}{r\cdot d}$.

Figure 3. The calculation of the inventory costs of NRFI items (*m = 3*)



Let us now summarize the areas:

$$H_{NFI}=(n-1)\cdot\frac{h_2}{2\cdot d}\cdot r\cdot Q_R^2+\frac{h_2}{d}\cdot(1-r)\cdot Q_R^2\cdot\sum_{i=1}^{n-1}i+\frac{h_2}{2\cdot r\cdot d}\cdot Q_R^2\cdot[n-(n-1)\cdot r]^2.$$

After some elementary calculation we have the equation b).

Example 1. Let $d = 1{,}000$, $r = 0.9$, $h_1 = \$\ 750$, $h_2 = \$\ 100$. Then for this data the inventory holding cost function is

$$H_{RFI}+H_{NRFI}=\frac{1}{10}\cdot m\cdot Q_P^2+\frac{11}{100}\cdot n\cdot Q_R^2+\frac{1}{900}\cdot n^2\cdot Q_R^2$$

4. Optimal procurement/repair cycle time

The fixed procurement and repair induction costs

$$F=m\cdot A_P+n\cdot A_R$$

The total average costs are

$$C(T,Q_P,Q_R,n,m,r)=\frac{F+H_{RFI}+H_{NFRI}}{T}=$$

$$=\frac{m\cdot A_P+n\cdot A_R+\frac{h_1}{2\cdot d}\cdot m\cdot Q_P^2+\frac{h_1}{2\cdot d}\cdot n\cdot Q_R^2+\frac{h_2}{2\cdot d}\cdot\left(n^2\cdot\frac{1-r}{r}+n\right)\cdot Q_R^2}{T}$$

Let now use the equations (1)

$$Q_P(T,m,r)=\frac{(1-r)\cdot d\cdot T}{m}$$
$$Q_R(T,n,r)=\frac{r\cdot d\cdot T}{n} \tag{2}$$

After substitution the economic order quantities in the cost function we obtain a simpler cost function:

$$C_1(T,n,m,r)=\frac{m\cdot A_P+n\cdot A_R}{T}+T\cdot\frac{d}{2}\cdot\left[h_1\cdot(1-r)^2\cdot\frac{1}{m}+(h_1+h_2)\cdot r^2\cdot\frac{1}{n}+h_2\cdot r\cdot(1-r)\right]$$

This function is convex in the cycle time then the necessary conditions of optimality are sufficient, as well. The optimal cycle time is

$$T^o(n,m,r)=\sqrt{\frac{2}{d}}\cdot\sqrt{\frac{m\cdot A_P+n\cdot A_R}{h_1\cdot(1-r)^2\cdot\frac{1}{m}+(h_1+h_2)\cdot r^2\cdot\frac{1}{n}+h_2\cdot r\cdot(1-r)}}$$

The simplified cost function is

$$C_2(n,m,r)=\sqrt{2\cdot d}\cdot\sqrt{(m\cdot A_P+n\cdot A_R)\cdot\left[h_1\cdot(1-r)^2\cdot\frac{1}{m}+(h_1+h_2)\cdot r^2\cdot\frac{1}{n}+h_2\cdot r\cdot(1-r)\right]}$$

or

$$C_2(n,m,r)=\sqrt{2\cdot d}\cdot\sqrt{A(r)\cdot\frac{m}{n}+B(r)\cdot\frac{n}{m}+C(r)\cdot m+D(r)\cdot n+E(r)}$$

where

$$A(r)=A_P\cdot(h_1+h_2)\cdot r^2,\quad B(r)=A_R\cdot h_1\cdot(1-r)^2,\quad C(r)=A_P\cdot h_2\cdot r\cdot(1-r),$$
$$D(r)=A_R\cdot h_2\cdot r\cdot(1-r),\quad E(r)=A_P\cdot h_1\cdot(1-r)^2+A_R\cdot(h_1+h_2)\cdot r^2$$

Example 2. Let *d = 1,000*, *r = 0.9*, h_1 *= \$ 200*, h_2 *= \$ 20*, A_P *= \$ 750*, A_R *= \$ 100*. Then for this data $A(0.9)=133{,}650$, $B(0.9)=200$, $C(0.9)=135$, $D(0.9)=180$, $E(0.9)=193{,}200$

5. The basic model of Schrady

Schrady has investigated the case with only one procurement batch *m = 1*. The cost function of this model is

$$C^S(n,r) = C_2(1,n,r) = \sqrt{2\cdot d}\cdot\sqrt{A(r)\cdot\frac{1}{n} + [B(r)+D(r)]\cdot n + [C(r)+E(r)]}$$

The optimal continuous solution for this case is

Lemma 2.

The solution of model of Schrady is

a) if $A_P\cdot(h_1+h_2)\cdot r^2 - A_R\cdot h_2\cdot r\cdot(1-r) - A_R\cdot h_1\cdot(1-r)^2 > 0$,
then $n^o(r) = \frac{r}{1-r}\cdot\sqrt{\frac{A_P}{A_R}}\cdot\sqrt{\frac{h_1+h_2}{h_1+h_2\cdot\frac{r}{1-r}}}$ and

$$C^S(n^o(r),r) = \sqrt{2\cdot d}\cdot\left[(1-r)\cdot\sqrt{A_P\cdot\left(h_1+h_2\cdot\frac{r}{1-r}\right)} + r\cdot\sqrt{A_R\cdot(h_1+h_2)}\right]$$

b) if $A_P\cdot(h_1+h_2)\cdot r^2 - A_R\cdot h_2\cdot r\cdot(1-r) - A_R\cdot h_1\cdot(1-r)^2 \le 0$,
then $n^o(r) = 1$ and

$$C^S(n^o(r),r) = \sqrt{2\cdot d}\cdot\sqrt{(A_P+A_R)\cdot\left[(h_1+h_2)\cdot r^2 + h_1\cdot(1-r)^2 + h_2\cdot r\cdot(1-r)\right]}$$

Proof. Let us investigate the function $C^S(n,r)$. This function is convex in *n*. The minimal value of the repair batch number is

$$n^o(r) = \sqrt{\frac{A(r)}{B(r)+D(r)}} = \frac{r}{1-r}\cdot\sqrt{\frac{A_P}{A_R}}\cdot\sqrt{\frac{h_1+h_2}{h_1+h_2\cdot\frac{r}{1-r}}}.$$

After substitution the optimal value of *n*, we have the condition a) of the lemma. If this number is smaller then one, then the cost function is mononuously increasing for all $n \geq 1$. This fact supports this condition b).

Remark 1. The function $F(r) = A_P\cdot(h_1+h_2)\cdot r^2 - A_R\cdot h_2\cdot r\cdot(1-r) - A_R\cdot h_1\cdot(1-r)^2$ is quadratic and monotomuously increasing between zero and one. Value $F(0) = -A_R\cdot h_1$ is negative and $F(1) = A_P\cdot(h_1+h_2)$ positive, so there exists a recovery rate r_2 for which $F(r_2) = 0$. Then the optimal batch number is equal to one for all $r\in[0,r_2]$ and it is greater than one for all $r\in(r_2,1]$.

Remark 2. The solution for the batch number is not always integer for all $r\in(r_2,1]$. If value $n^o(r)$ is integer then the problem is solved. Let us now assume that $n^o(r)$ is not integer. Let

$\underline{n}(r)=int(n^o(r))$ denote the maximal integer not greater than $n^o(r)$ and $\overline{n}(r)=int(n^o(r))+1$ the minimal integer not smaller than $n^o(r)$. The optimal integer solution can be determined from the following relation

$$n_i^o(r)=\arg\min\{C^S(\underline{n}(r)),\ C^S(\overline{n}(r))\}.$$

Theorem 1.

The optimal continuous the cycle time and order quantities of model of Schrady are

$$T^o(r)=\begin{cases}\sqrt{\frac{2}{d}}\cdot\sqrt{\frac{A_P+A_R}{h_1\cdot(1-r)^2+(h_1+h_2)\cdot r^2+h_2\cdot r\cdot(1-r)}} & r\in[0,r_2]\\ \sqrt{\frac{2}{d}}\cdot\sqrt{\frac{A_P}{h_1\cdot(1-r)^2+h_2\cdot r\cdot(1-r)}} & r\in(r_2,1]\end{cases}$$

$$Q_P^o(r)=\begin{cases}\sqrt{\frac{2\cdot d\cdot(A_P+A_R)\cdot(1-r)^2}{h_1\cdot(1-r)^2+(h_1+h_2)\cdot r^2+h_2\cdot r\cdot(1-r)}} & r\in[0,r_2]\\ \sqrt{\frac{2\cdot d\cdot A_P\cdot(1-r)}{h_1\cdot(1-r)+h_2\cdot r}} & r\in(r_2,1]\end{cases}$$

and

$$Q_R^o(r)=\begin{cases}\sqrt{\frac{2\cdot d\cdot(A_P+A_R)\cdot r^2}{h_1\cdot(1-r)^2+(h_1+h_2)\cdot r^2+h_2\cdot r\cdot(1-r)}} & r\in[0,r_2]\\ \sqrt{\frac{2\cdot d\cdot A_R}{h_1+h_2}} & r\in(r_2,1]\end{cases}$$

Proof. If $r\in[0,r_2]$, i.e. the optimal repair batch number is one, then after substitution we have the optimal cycle and order quantities. To determine the other case, we use the following relation

$$T^o(n^o(r),r)=\sqrt{\frac{2}{d}}\cdot\frac{n^o(r)}{r}\cdot\sqrt{\frac{A_R}{h_1+h_2}}\ .$$

Substituting the optimal repair batch number and cycle time in equations (2), we get the results of the theorem.

Schrady in his paper has not analyzed those cases, for which the optimal batch number is even one. In this formulation we have shown that the solution supplied by Schrady is limited to the case for $r\in(r_2,1]$. The method poposed in this paper has the same result for the economic order quantities, as obtained by Schrady. The optimal cycle time and economic order quantities for the integer batch number can be calculated with substitution and with some elementary operations.

Example 3. Let as in Ex. 2. $d = 1{,}000$, $r = 0.9$, $h_1 = \$ 200$, $h_2 = \$ 20$, $A_P = \$ 750$, $A_R = \$ 100$. Then for this data the optimal continuous solution and the switching point r_2 are $r_2 = 0.2316$ and $n^o = 18.754,\ m^o = 1,\ T^o = 0.628\ years,\ Q_P^o = 62.828,\ Q_R^o = 30.151,\ C^S = \$8{,}357.4$.

6. The optimal number of repair and procurement batches

To minimize the costs in dependence on the batch numbers we apply an auxiliary problem (meta-model). The problem is

$$C_2(m,n,r) = \sqrt{2\cdot d}\cdot\sqrt{A(r)\cdot\frac{m}{n} + B(r)\cdot\frac{n}{m} + C(r)\cdot m + D(r)\cdot n + E(r)} \rightarrow min$$

subject to

$m \geq 1,\quad n \geq 1$. This problem was extensively studied in papers [1-5]. Based on the mentioned papers we examine the continuous solution of this model.

Theorem 2.

There is three cases of optimal solutions $(n(r), m(r))$ and the minimum cost expressions $C_3(r)$ in dependence on the return rate for this problem

(i) $$A_P\cdot(h_1+h_2)\cdot r^2 - A_R\cdot h_1\cdot(1-r)^2 + A_P\cdot h_2\cdot r\cdot(1-r) < 0$$

$$(n(r), m(r)) = \left(1,\ \sqrt{\frac{A_R}{A_P}}\cdot\frac{1-r}{r}\cdot\sqrt{\frac{h_1}{h_1+\frac{h_2}{r}}}\right)$$

$$C_3(r) = \sqrt{2\cdot d}\cdot\left\{r\cdot\sqrt{A_R\cdot(h_1+h_2)} + \sqrt{A_P\cdot(1-r)\cdot[h_1\cdot(1-r)+h_2\cdot r]}\right\}$$

(ii) $$0 \leq A_P\cdot(h_1+h_2)\cdot r^2 - A_R\cdot h_1\cdot(1-r)^2 + A_P\cdot h_2\cdot r\cdot(1-r) \leq (A_R+A_P)\cdot h_2\cdot r\cdot(1-r)$$

$$(n(r), m(r)) = (1, 1)$$

$$C_3(r) = \sqrt{2\cdot d}\cdot\sqrt{(A_P+A_R)\cdot\left[(h_1+h_2)\cdot r^2 + h_1\cdot(1-r)^2 + h_2\cdot r\cdot(1-r)\right]}$$

(iii) $$A_P\cdot(h_1+h_2)\cdot r^2 - A_R\cdot h_1\cdot(1-r)^2 + A_P\cdot h_2\cdot r\cdot(1-r) > (A_R+A_P)\cdot h_2\cdot r\cdot(1-r)$$

$$(n(r), m(r)) = \left(\sqrt{\frac{A_P}{A_R}}\cdot\frac{r}{1-r}\cdot\sqrt{\frac{h_1+h_2}{h_1+h_2\cdot\frac{r}{1-r}}},\ 1\right)$$

$$C_3(r) = \sqrt{2\cdot d}\cdot\left\{(1-r)\cdot\sqrt{A_P\cdot h_1} + \sqrt{A_R\cdot r\cdot[h_1\cdot r + h_2]}\right\}$$

It is easy to see that the three regions for the optimal solution in dependence on the return rate are not intersected. So we can calculate the values r_1 and r_2 ($r_1 < r_2$) for which either the procurement batch or the repair batch is equal to one, but the other batch number is greater than one. Between these values both of the batch numbers are equal to one.

Example 4. Let as in Ex. 3. $d = 1{,}000$, $h_1 = \$ 200$, $h_2 = \$ 20$, $A_P = \$ 750$, $A_R = \$ 100$. Then for this data $r_1 = 0.2341$ and $r_2 = 0.2616$. Let now substitute $r = 0.9$ in the optimal solution. Then the optimal values are as calculated in Ex. 3.

The optimal procurement and repair batch sizes and the cycle times of the model are in dependence on the return rate:

Theorem 3.

The order quantities and cycle times are

(i) $r \in [0, r_1)$

$$T(r) = \sqrt{\frac{2}{d}} \cdot \sqrt{\frac{A_R}{r \cdot (h_1 \cdot r + h_2)}},$$

$$Q_P(r) = \sqrt{2 \cdot d} \sqrt{\frac{A_P}{h_1}},$$

$$Q_P(r) = \sqrt{2 \cdot d} \sqrt{\frac{A_R \cdot r}{h_1 \cdot r + h_2}}.$$

(ii) $r \in [r_1, r_2]$

$$T(r) = \sqrt{\frac{2}{d}} \cdot \sqrt{\frac{A_P + A_R}{h_1 \cdot \left[(1-r)^2 + r^2\right] + h_2 \cdot r}},$$

$$Q_P(r) = \sqrt{2 \cdot d} \sqrt{\frac{(A_P + A_R) \cdot (1-r)^2}{h_1 \cdot \left[(1-r)^2 + r^2\right] + h_2 \cdot r}},$$

$$Q_P(r) = \sqrt{2 \cdot d} \sqrt{\frac{(A_P + A_R) \cdot r^2}{h_1 \cdot \left[(1-r)^2 + r^2\right] + h_2 \cdot r}}.$$

(iii) $r \in (r_2, 1]$

$$T(r) = \sqrt{\frac{2}{d}} \cdot \sqrt{\frac{A_P}{h_1 \cdot (1-r)^2 + h_2 \cdot r \cdot (1-r)}},$$

$$Q_P(r) = \sqrt{2 \cdot d}\sqrt{\frac{A_P \cdot (1-r)}{h_1 \cdot (1-r) + h_2 \cdot r}},$$

$$Q_P(r) = \sqrt{2 \cdot d}\sqrt{\frac{A_R}{h_1 + h_2}}.$$

The proof is easy, we must substitute the continuous batch numbers in the order quantities and cycle times.

Example 5. Let as in Ex. 3. $d = 1,000$, $r = 0.05$, $h_1 = \$ 200$, $h_2 = \$ 20$, $A_P = \$ 750$, $A_R = \$ 100$. Then for this data the minimal cost for the basic model: $C^S(0.05) = 17,589.8$ and the nimimal cost for the generalized model $C(0.05) = 17,002.2$. This means a cost saving of *3.5* percent of the total EOQ related costs.

7. Conclusion

In this paper we have reformulated and solved the model of Schrady. We have shown that for smaller recovery rate it gives a better solution if the procurement batch number is greater than one and on the basis of model of Schrady we can obtain a more effective solution for higher return rate. This result can be interpreted as a generalization of model of Schrady for the case of more than one procurement batch.

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